Lecture 1: Reverse engineering the brain

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- Asking how something works is fundamentally a question about how it serves a function.
- The heart pumps, the stomach digests, the brain thinks.
- The real question: *How does the brain produce thought?*
- **Thought is computation**—the manipulation of representations for some purpose.

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- Mental computations are purposeful manipulations of representations.

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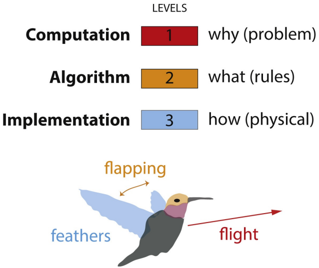
1. **Computational level**: What is the problem being solved by the system?
2. **Representational/algorithmic level**: How is the problem solved algorithmically?
3. **Implementation level**: How is the algorithm realized physically?

Why are the levels useful?

Marr: "Trying to understand perception by studying only neurons is like trying to understand bird flight by studying only feathers: It just cannot be done."



[Krakauer et al. 2017]

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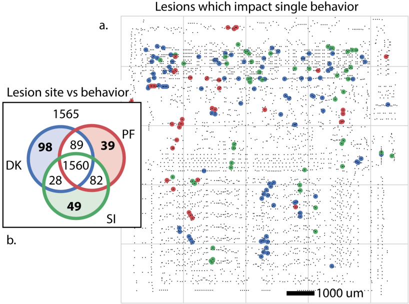
2. **Methodological**: Thinking like an engineer is often a good starting place for building models.

How far can we go with a hardcore bottom-up approach?

Lesioning a microprocessor to identify the functions of individual transistors: an exercise in futility?



[Jonas & Kording 2017]

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2. Engineer algorithmic solutions to these problems.
3. Model how the brain could implement the algorithmic solutions under biological constraints.

Study question

What are the advantages and disadvantages of the reverse engineering approach?

The computational level: statistical decision problems

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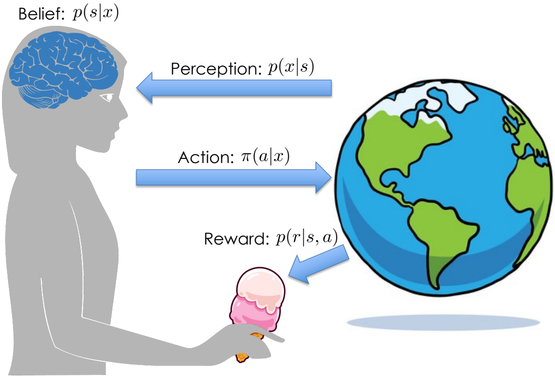
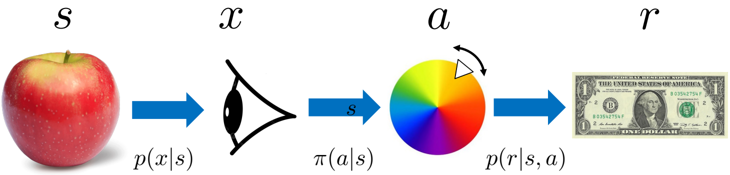
- On further inspection, these all appear to be variations on one kind of problem: a statistical decision problem.
- This is general enough to encompass many (all?) specific functions carried out by the brain.

Decision theory



Example



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- This emphasizes the fact that utility is distinct from nominal quantities like dollars, number of calories, etc. Utility is internally generated.

Bayesian decision theory

- Key idea: maximize *expected utility* $\bar{u}(\pi) = \mathbb{E}[u(r)|\pi]$ given beliefs $p(s|x)$ about the hidden state of the world:

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- Expected utility is how much utility an agent believes it will gain under policy π, averaging over these sources of randomness:

$$\bar{u}(\pi) = \sum_x p(x) \sum_a \pi(a|x) \sum_s p(s|x) \sum_r p(r|s,a)u(r).$$

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- The likelihood, $p(x|s)$, expresses how well a hypothetical state "fits" the data.

Study question

If priors are subjective, are Bayesian theories unfalsifiable?

Why be Bayesian?

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  - Conjunction: AB
  - Disjunction $A + B - AB$
  - Negation: $1 - A$
- Truth values are known with certainty (Boolean operations always yield values of 0 or 1). What if you're unsure? Is there a "soft" version that correctly represents and propagates a measure of "plausibility"?

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- We'd also like it to recover Boolean logic as a special case when plausibility is maximal or minimal (corresponding to complete certainty).
- Cox's Theorem: only probabilities updated according to Bayes' rule satisfy these requirements.
- Thus, Bayesian probability theory can be viewed as a natural extension of Boolean logic.

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- Complete Class Theorem: every admissible policy corresponds to a Bayesian policy for some prior.
- Thus, Bayesian decision theory is in a sense inevitable for a decision maker who wants to avoid being dominated.

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- If instead the agent makes bets using plausibilities that violate the axioms of probability, then it's possible to construct a bet that they will accept and yet they will be guaranteed to lose money.
- Thus, there is a financial incentive to be Bayesian.

The algorithmic level

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- These constraints delimit what kinds of algorithms are realizable.

Complexity, efficiency, tractability

We can characterize the requirements of an algorithm along several dimensions:

1. **Time complexity**: how much computation is required?
2. **Space complexity**: how much memory is required?
3. **Sample complexity**: how much data are required?

If complexity cannot be expressed as a polynomial function of the input size N, an algorithm is considered inefficient. A problem for which no efficient algorithm exists is intractable.

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- Computing the normalizing constant for Bayes' rule then requires summing over K^N possible configurations.
- Example: x corresponds to images and s corresponds to the set of N objects in a scene, each of which could belong to K possible categories.
- Enumerating all possible states is inefficient because N appears in the exponent.

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- Exponential complexity frequently arises in high-dimensional problems where some computation requires exhaustive coverage of the space—the *curse of dimensionality*.

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- Only reverse engineer the brain's efficient solutions to tractable problems.
- Focus on algorithms with polynomial complexity that have been shown to work in practice.
- Identify behavioral and neural signatures of these algorithms, investigate how they could be implemented with neural machinery.

Resource rationality

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- The resource-rational policy optimizes expected utility subject to the resource capacity limit:

$$\pi^* = \operatorname*{argmax}_{\pi:\, c(\pi) \leq \mathcal{C}} \bar{u}(\pi).$$

where $c(\pi)$ is the amount of resources consumed by implementing policy π.

Study question

How is it possible to find optimal resource-constrained policies when the policy search is itself resource-constrained? And doesn't this threaten an infinite regress, where each optimization is nested within an even more difficult optimization problem?

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- The brain's elementary computing units are neurons.
- Each neuron implements a relatively simple computation; wiring up many neurons together makes complex computation possible.

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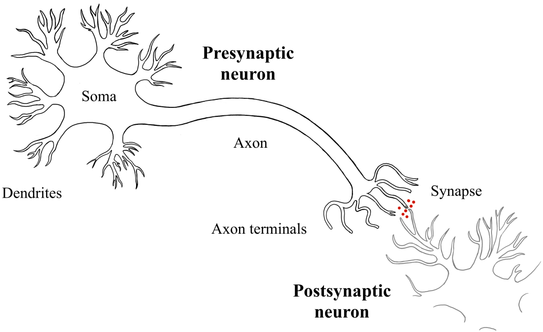

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- Maintaining a reliable response to inputs is also metabolically expensive.
- Maintenance of synaptic weights is metabolically expensive.
- The brain should economize on the number of neurons, their average firing rate, the reliability of firing, and the number/strength of connections between neurons.

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- The purpose: Bayesian decision theory.
- The representations: probabilistic beliefs, utilities, and costs.
- The implementational primitives: networks of neurons connected by plastic synapses.